Mathematics

INSTRUCTIONS TO CANDIDATES

• Maximum marks: 44
• Working time: 60 minutes
• Number of questions: 12
• Don’t use a calculator, please!
• You are advised to show all working where possible. Where an answer is wrong some marks may be given for correct method provided this is shown by written working. For the solution of can use also clean pages of paper. Please, don’t forget to mark the number of the question.

• Trigonometric values:
  \[
  \begin{align*}
  \sin \frac{\pi}{6} &= \cos \frac{\pi}{3} = \frac{1}{2} \\
  \sin \frac{\pi}{3} &= \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\
  \sin \frac{\pi}{4} &= \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \\
  \tan \frac{\pi}{3} &= \cot \frac{\pi}{6} = \sqrt{3}
  \end{align*}
  \]
1. Find the relation between $p$ and $q$ if $px^2 + qx + 1 = 0$ has equal roots. 

Answer: 

2. A rectangle has area $(6 - \sqrt{3}) \text{ cm}^2$ and the length of one of its sides is $(2 + \sqrt{3}) \text{ cm}$. Find the length of the other side in the form $(a + b\sqrt{c})$. 

Answer: 

3. Find exactly the values of $\sin \alpha$, $\tan \alpha$ if $\cos \alpha = -\frac{1}{4}$ and $\alpha$ is an obtuse angle. 

Answer: 

4. The square of Richard’s age is equal to his mother’s age. When he is twice as old as he is now, she will be $7/2$ times as old as he will be. Find the ages of Richard and his mother. 

Answer: 

5. The sides of a rectangle are in the ratio $2 : 3$. The diagonal is of length $26$ cm. Find the perimeter.

Answer: 

6. (a) Find $p$, $q$ and $r$ such that $5x^2 - 2x + 1 = p(x - q)^2 + r$.
(b) Find the minimum value of $5x^2 - 2x + 1$ and the value of $x$ for which it occurs.

Answers: (a) ........................................ (b) ........................................
7. (a) In how many ways can 7 students be arranged in a row?
(b) In how many ways can 7 students be arranged in a circle?
(c) In how many ways can this be done (in a circle) if the tallest girl and the tallest boy must be next to each other?

Answers: (a) ...........................................  (b) ...........................................
(c) ...........................................

8. Find the range for these functions. If the functions are periodic find the period in radians.
   (a) $y = |\cos x|$  
   (b) $y = \sin \frac{x}{\pi}$  
   (c) $y = \tan|x|

Answers:  (a) ...........................................  (b) ...........................................
(c) ...........................................

9. The area of the triangle is $12 \text{ cm}^2$. The angle $ACB = 30^\circ$, $AC = (x+2) \text{ cm}$, $BC = x \text{ cm}$. Calculate the value of $x$.

Answer: .............................................
10. Solve the inequality: $|4 - x| > 1 + |x + 1|$

Answer: 

11. Find the exact solutions of the equation $2 \cos^2 x = \cos|x|$, for $-\pi \leq x \leq 2\pi$.

Answer:
12. Given that \( f(x) = \frac{2x + 1}{1 - x} \)

(a) Find domain and range of \( f(x) \).
(b) Sketch the graphs of \( f(x) \), \( f^{-1}(x) \) on the same axes.
(c) Solve the equation \( f^{-1}(x) = f(x) \).

Answers: 

(a) .....................................

(b) .....................................

(c) .....................................